TRINITY COLLEGE

Year 12 Mathematics Specialist Units 3, 4
Test 5 2021

Section 1 Calculator Free
Rates of Change and Differential Equations

STUDENT'S NAME ______________________________

DATE: Monday 23 August **TIME:** 15 minutes **MARKS**: 16

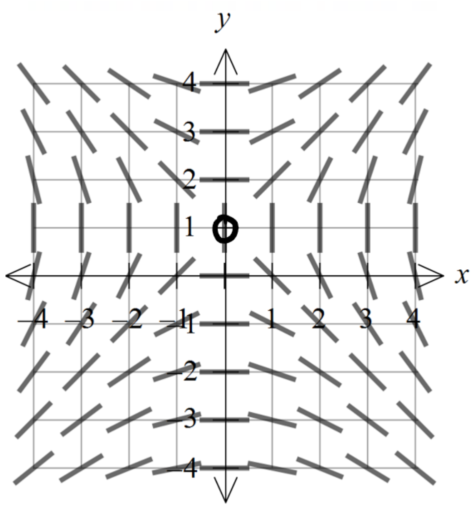
INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser, Formula Booklet

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

---

1. (3 marks)

Consider the following slope field



(a) On the diagram above, draw the solution curve that passes through:

(i) $(1,0)$ [1]

(ii) $(-2,1)$ [1]

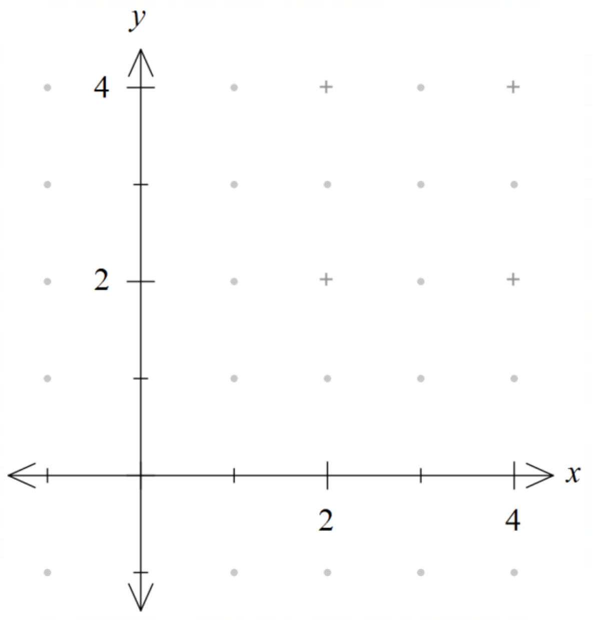
(b) Determine the equation of the slope field. [1]

2. (3 marks)

Consider the slope field given by $\frac{dy}{dx} = \frac{y-x}{x}$

(a) Calculate the value of the slope field at the point $(2,0)$ [1]

(b) Draw the slope field on the Cartesian plane below. [2]



3. (6 marks)

A particle travels in a straight line so that its velocity v cm per second and displacement x cm are related by the equation

$$v = -0.2x$$

(a) Determine the acceleration a in terms of its displacement x. [2]

It is known that the initial displacement of the particle is $x = 4cm$.

(b) The particle has a displacement of 2 cm at time $t = a \ln b$. Determine the values of a and b. [4]

4. (4 marks)

Solve the differential equation $\frac{dy}{dx} = \frac{e^{2y}}{x}$ for y in terms of x subject to the initial condition $(e, -1)$

Year 12 Mathematics Specialist Units 3, 4
Test 5 2021

Section 2 Calculator Assumed
Rates of Change and Differential Equations

STUDENT'S NAME ______________________________

DATE: Monday 23 August **TIME:** 35 minutes **MARKS**: 34

INSTRUCTIONS:

Standard Items:	Pens, pencils, drawing templates, eraser
Special Items:	Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

This page has been left blank intentionally

5. (8 marks)

Newton's law of cooling states that the rate of temperature decrease is proportional to the difference between the temperature of a body and its surroundings. This can be expressed as

$$\frac{dT}{dt} = -k\left(T - T_s\right) \text{ where } T_s \text{ is the temperature of the surroundings.}$$

After 10 minutes in Lucas' room, his tea has cooled to 40° Celsius from 100° Celsius. The room temperature is 25° Celsius.

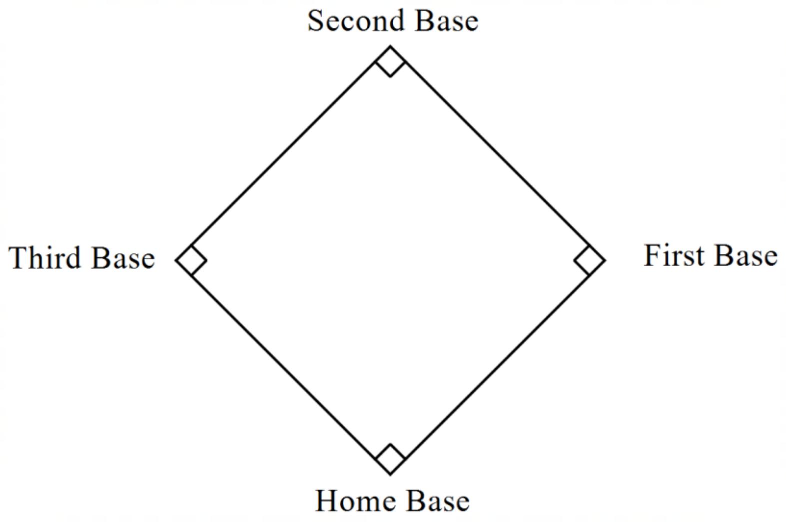
(a) Determine an equation for T, the temperature of the tea after t minutes. [4]

(b) How much longer will it take for Lucas' tea to cool to 35°? [3]

(c) How long will it take for Lucas' tea to cool to 25°? Explain. [2]

6. (7 marks)

During the annual baseball maths, Patrick hits the ball and runs from home plate to first base, while Tommy runs from first base to second base. The bases are arranged in a square with side length of 30 m.



At the moment when Patrick is halfway to first base, Tommy is two thirds of the way from first base to second base. At this moment, Patrick is running at a speed of 8 m/s and Tommy is running at a speed of 10 m/s.

What rate is the area of the right triangle formed by Patrick, Tommy and first base changing? Is the area increasing or decreasing?

7. (11 marks)

A rumour that Trinity College plans to build student parking at Waterbank begins to spread around the College. There is a combined total of 1500 students and staff at Trinity College.

Two hundred and fifty people know of this rumour at 7 am one morning.

Let $N(t)$ be the number of people at the College who have heard the rumour at t hours after 7 am. It is found that the rate at which the rumour spreads is given by

$$\frac{dN}{dt} = k\,N\,(1500 - N)\,.$$

At 7 am the rumour was spreading at a rate of 50 people per hour.

(a) Show that $k = 0.00016$ [2]

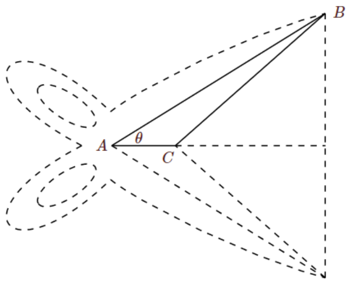
(b) At 12 pm approximately 600 had heard the rumour. Using the increments formula, determine the approximate number of people that learn of this rumour between 12 pm and 12:10 pm. [3]

(c) Determine an equation for $N(t)$ in the form $N(t) = \frac{a}{1+be^{ct}}$ [2]

(d) Determine the rate and time, to the nearest minute, when the rumour spreads at the fastest rate? [4]

8. (8 marks)

The **two** blades of a pair of scissors are fastened at the point A as shown in the figure below. Let a denote the distance from A to the top of the blade (the point B). Let β denote the angle at the tip of the blade that is formed by the line $\overline{AB}$ and the bottom edge of the blade, line $\overline{BC}$, and let θ denote the angle between $\overline{AB}$ and the horizontal. Suppose that a piece of paper is cut in such a way that the centre of the scissors at A is fixed, and the paper is also fixed. As the **blades** are closed, the distance x between A and C increases, cutting the paper.



(a) Show that $x = \dfrac{a \sin \beta}{\sin(\theta + \beta)}$ [3]

(b) Determine an expression for $\dfrac{dx}{dt}$. [2]

(c) Suppose that the distance a is 20 cm, the angle β is 5°, and that θ is decreasing at 50 deg/sec. At the instant when $\theta = 30^\circ$, determine the rate at which the paper is being cut. [3]

Sources

[6.2 Related Rates (whitman.edu)]

[relatedrates.pdf (arizona.edu)]

[ProblemSet.pdf (umich.edu)]